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Heat Transfer on Steady MHD rotating flow through porous medium in a parallel plate channel

Dr. G. Prabhakara Rao, M. Naga Sasikala, Dr. P. Gayathri
Lecturer in Mathematics, SVGM Govt. Degree College, Kalyandurg, Anantapur, A.P. India
Lecturer in Mathematics, K. S. N. Government Degree College Anantapur (Women)-Andhra Pradesh-India
Lecturer in Mathematics, Government Degree College Rayachoty,YSR Dist-Andhra Pradesh-India

Abstract
We discussed the combined effects of radiative heat transfer and a transverse magnetic field on steady rotating flow of an electrically conducting optically thin fluid through a porous medium in a parallel plate channel and non-uniform temperatures at the walls. The analytical solutions are obtained from coupled nonlinear partial differential equations for the problem. The computational results are discussed quantitatively with the aid of the dimensionless parameters entering in the solution.

Keywords: steady hydro magnetic flows, three dimensional flows, parallel plate channel, porous medium, radiative heat, optically thin fluid.

I. Introduction

The study of flow in rotating porous media is motivated by its practical applications in geophysics and engineering. Among the applications of rotating flow in porous media to engineering disciplines, one can find the food processing industry, chemical process industry, centrifugation filtration processes and rotating machinery. Also the hydro dynamic rotating flow of electrically conducting viscous incompressible fluids has gained considerable attention because of its numerous applications in physics and engineering. In geophysics, it is applied to measure and study the positions and velocities with respect to a fixed frame of reference on the surface of earth, which rotate with respect to an inertial frame in the presence of its magnetic field. The subject of geophysical dynamics now-a-days has become an important branch of fluid dynamics due to the increasing interest to study environment. In Astrophysics, it is applied to study the stellar and solar structure, inter planetary and inter stellar matter, solar storms etc. In engineering, it finds its application in MHD generators, ion propulsion, MHD bearings, the three-dimensional free convective channel flow MHD pumps, MHD boundary layer control of re-entry vehicles etc. The flow of fluids through porous media are encountered in a wide range of engineering and industrial applications such as in recovery or extraction of crude oil, geothermal systems, thermal insulation, heat exchangers, storage of nuclear wastes, packed bed catalytic reactors, atmospheric and oceanic circulations. Comprehensive literature on buoyancy induced flows can be found Nield and Bejan (2006). The study of flow of electrically conducting fluid, the so-called magneto hydro dynamics (MHD) has a lot of attention due to its diverse applications. In astrophysics and geophysics, it is applied to the study of stellar and solar structures, interstellar matter, and radio propagation through the ionosphere. In engineering, it finds its application in MHD pumps, MHD bearings, nuclear reactors, geothermal energy extraction and in boundary layer control in the field of aerodynamics. A survey of MHD studies could be found in Crammer and Pai (1973); Moreau (1990). For example, Raptis *et al.* (1982) analyzed the problem of hydro magnetic free convection flow through a porous medium between two parallel plates; while Kearsley (1994) studied problem of steady state Couette flow with viscous heating. Makinde and Osalusi (2006) considered a MHD steady flow in a channel with slip at permeable boundaries. More recently, many researchers have focused attention on MHD applications where the operating temperatures are high. For example, at high temperatures attained in some engineering devices, gas can be ionized and so become electrically conducting. The ionized gas or plasma can be made to interact with the magnetic field and alter the heat and friction characteristics of the system. It is important to study the effect of the interaction of magnetic field on the temperature distribution and heat transfer when the fluid is not only electrically conducting but also when it is capable of emitting and absorbing thermal radiation. Heat transfer by thermal radiation is important when we are concerned with space technology applications and in power engineering. Thus, Grief *et al.* (1971) obtained an exact solution for the problem of laminar convective flow in a vertical heated channel within the optically thin limit of Cogley *et al.* (1968). Makinde and Mhone (2005) investigated the effect of thermal radiation on MHD oscillatory flow in a channel filled with saturated porous medium and non-uniform wall temperatures. Kumar *et al.* (2010) considered the problem of unsteady MHD periodic flow of viscous fluid through a planar channel in porous medium using perturbation techniques. Narahari (2010) studied the effects of thermal radiation and free convection currents on the unsteady Couette flow between two vertical parallel plates with constant heat flux at one boundary. Israel Cookey and Nwaigwe (2010) considered unsteady MHD flow of a radiating fluid over a vertical moving heated porous plate with time -dependent suction. Recently Israel Cookey.C, *et al.* (2010) investigated the combined effects of thermal radiation and transverse magnetic field on steady flow of electrically conducting optically thin fluid through a horizontal channel filled with saturated porous medium and non-uniform wall temperatures. K.D.Singh & Reena Pathak (2012) discussed the effects of hall current and rotation on MHD free convection flow in a vertical rotating channel filled with porous medium. In this paper, we discussed the combined effects of radiative heat transfer and a transverse magnetic field on steady rotating flow of an electrically conducting optically thin fluid through a porous medium in a parallel plate channel and non-uniform temperatures at the walls.

II. Formulation and Solution of the Problem

We consider the buoyancy induced steady flow of an electrically conducting optically thin fluid bounded by two parallel plates filled with saturated porous medium under the influence of a transverse uniform magnetic field of strength B_0. The lower plate which is on $z = 0$ is maintained at temperature $T = T_0$ and the upper plate at $z = h$ is maintained at temperature $T = T_1$. A Cartesian co-ordinate system with x-axis oriented horizontally along the centre of the channel is introduced. The $z-$axis is taken perpendicular to the planes of the plates is the axis of rotation and the entire system rotates about this axis with uniform angular velocity Ω as shown in the figure 1.

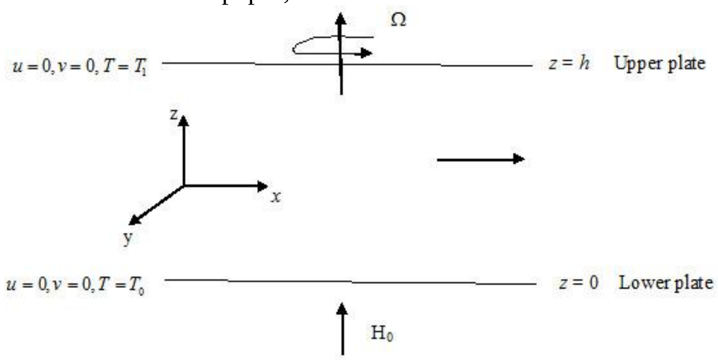

Fig.1 Physical configuration of the problem

We are assuming a Boussinesq incompressible fluid model and taking into consideration the radiative heat flux, the governing equations for the unsteady magneto hydrodynamic flow of viscous incompressible fluid through a porous medium bounded between two parallel plates in presence of thermal radiation,

$$-2\Omega v = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma\mu_e^2 H_0^2}{\rho}u - \frac{\nu}{k}u + g\beta_T(T - T_0) \tag{2.1}$$

$$2\Omega u = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{\partial^2 v}{\partial z^2} - \frac{\sigma\mu_e^2 H_0^2}{\rho}v - \frac{\nu}{k}v \tag{2.2}$$

$$\frac{K_T}{\rho C_p}\left(\frac{\partial^2 T}{\partial z^2} - \frac{1}{K_T}\frac{\partial q}{\partial z}\right) = 0 \tag{2.3}$$

Where u and v are the velocity components along the x and y directions, *p* the pressure, *T* the temperature, g the gravitational acceleration, *q* the radiative heat flux, β_T the coefficient of thermal expansion, ν the coefficient of kinematic viscosity, σ the electric conductivity, μ_e the magnetic permeability, H_o is the applied magnetic field ρ density of the fluid, c_p the specific heat capacity at constant pressure, K_T the thermal conductivity and *k* the permeability of the porous medium.

Since the plates extends to infinity along *x* and *y* directions, all the physical quantities except the pressure depend on *z* alone, and hence the respective equations of continuity are trivially satisfied.

The boundary conditions are

$$u = 0,\ v = 0,\ T = T_0 \quad \text{on} \quad z = 0 \tag{2.4}$$

$$u = 0,\ v = 0,\ T = T_1 \quad \text{on} \quad z = h \tag{2.5}$$

Let $F = u + iv$, $\xi = x - iy$

Now combining equations (2.1) and (2.2), we obtain

$$-2\Omega F = -\frac{1}{\rho}\frac{\partial p}{\partial \xi} + \nu \frac{\partial^2 F}{\partial z^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho} F - \frac{\nu}{k} F + g\beta_T (T - T_0) \tag{2.6}$$

Corresponding boundary conditions are

$$F = 0, T = T_0 \quad \text{on} \quad z = 0 \tag{2.7}$$

$$F = 0, T = T_1 \quad \text{on} \quad z = h \tag{2.8}$$

We assume that the temperatures T_0, T_1 of the walls are high enough to induce radiative heat transfer. Following Cogley *et al.* (1968) and assuming that the fluid is optically thin with relatively low density, then

$$\frac{\partial q}{\partial z} = 4\alpha^2 (T - T_0) \tag{2.9}$$

where α is the mean radiation absorption coefficient.

In order to simplify the problem, we introduce the following non-dimensional variables and parameters.

$$z^* = \frac{z}{h}\ ,\ u = \frac{u^*}{U}, v = \frac{v^*}{U},\ q = \frac{q^*}{U}\ ,\ \theta = \frac{T - T_0}{T - T_1}$$

Where *U* is the mean velocity.

Making use of non-dimensional variables, the governing equations (2.3) and (2.6) reduces to

$$\frac{\partial^2 F}{\partial z^2} - (M^2 + D^2 + 2iE)F = -P - Gr\,\theta \tag{2.10}$$

$$\frac{\partial^2 \theta}{\partial z^2} - R^2 \theta = 0 \tag{2.11}$$

Corresponding boundary conditions are

$$F = 0, \theta = 0 \quad \text{on} \quad z = 0 \tag{2.12}$$

$$F = 0, \theta = 1 \quad \text{on} \quad z = 1 \tag{2.13}$$

Where,

$M^2 = \dfrac{\sigma \mu_e^2 H_0^2 h^2}{\rho \nu}$ is the Hartmann number (Magnetic field parameter),

$D^2 = \dfrac{k}{h^2}$ is the porosity parameter,

$E = \dfrac{\Omega h^2}{\nu}$ is the Rotation parameter,

$Gr = \dfrac{gB_T h^2 (T_0 - T_1)}{\nu U}$ is the Grashof number,

$R^2 = \dfrac{4\alpha^2 h^2}{K_T}$ is the Radiation parameter

$P = \frac{\partial p}{\partial \xi}$ is the applied pressure gradient.

The mathematical formulation of the problem is now complete and embodies the solution of equations (2.10) and (2.11) subject to conditions (2.12) and (2.13). The problem posed in equations (2.10) and (2.11) are coupled nonlinear partial differential equations. The closed form solutions are herein deduced. We begin by solving the energy equation (2.11) since it is uncoupled and then advance a solution for the flow velocity. The solutions to the temperature, $\theta(z)$ and velocity, $F(z)$ expressions are given by

$$F(z) = \left[P + \frac{Gr}{\lambda^2 - R^2} \right] \left(\frac{Sinh(Rz)}{Sinh(R)} - \frac{Sinh(\lambda z)}{Sinh(\lambda)} \right) \tag{2.14}$$

$$\theta(z) = \frac{Sinh(Rz)}{Sinh(R)} \tag{2.15}$$

Where, $\lambda^2 = M^2 + D^2 + 2iE$

Skin friction: Using Equation (2.14), the skin-friction or the shear stress at the upper wall of the channel in non-dimensional form, is given by

$$\tau = -\left(\frac{\partial F}{\partial z} \right)_{z=1} = -\left[P + \frac{Gr}{\lambda^2 - R^2} \right] \left(\frac{R\,Cosh(Rz)}{Sinh(R)} - \frac{\lambda\,Cosh(\lambda z)}{Sinh(\lambda)} \right) \tag{2.16}$$

Nusselt Number: From the temperature profile (equation (2.15)), the rate of heat transfer across the channel in non-dimensional form is given by

$$Nu = -\left(\frac{\partial \theta}{\partial z} \right)_{z=1} = -\frac{R\,Cosh(R)}{Sinh(R)} \tag{2.17}$$

III. Results and Discussion

In the preceding section, we have formulated and solved the problem of steady hydro magnetic flow of a radiating viscous fluid through a rotating parallel plate channel filled with porous materials. The complete expressions for the velocity, $u(z)$ and temperature, $\theta(z)$ profiles as well as the skin friction, τ and the heat transfer rate, Nu are given in equations (2.14) – (2.17). In order to understand the physical situation of the problem and hence the manifestations of the effects of the material parameters entering into the solution of problem, we have computed the numerical values of the velocity, temperature, skin friction and the rate of heat transfer. The computational results are presented in Figures (2 – 13) and Tables (1 – 3).

It is evident from Figures (2 & 3) that, the magnitude of the velocity component u increases and the velocity component v decreases with the increase of Hartmann number M. This is because of the reason that effects of a transverse magnetic field on an electrically conducting fluid gives rise to a resistive type force (called Lorentz force) similar to drag force and upon increasing the values of M increases the drag force which has tendency to slow down the motion of the fluid. The resultant velocity decreases with increasing the intensity of the magnetic field. The variation of the velocity profile on permeability of the porous medium D is shown in Fig. (4 & 5). It is observed that in the rotating channel the velocity u increases and v decreases with increasing D. It is expected physically also because the resistance posed by the porous medium to the decelerated flow due to rotation with increasing permeability D which leads to increase in the velocity. Lower the permeability of the porous medium lesser the fluid speed is observed in the entire channel. The resultant velocity increases with increasing the porosity parameter D. The variation of velocity profiles under the influence of the rotation parameter E is observed from Figures (6 & 7). Both the velocity components u & v increase when E is increased. The resultant velocity also increases with increasing the rotation parameter E. The variations of the velocity profiles with the Grashof number Gr are shown in Fig.(8 & 9). For small rotations (E=1), the velocity increases with the increasing Grashof number. The maximum of the velocity profiles shifts towards right half of the channel due to the greater buoyancy force in this part of the channel due to the presence of lower plate. For large rotation (E=4), the Grashof number has parabolic in nature on the velocity profiles in the right half and the left half of the channel. In the upper half there lies upper plate at z = 0 and heat is transferred from the upper plate to the fluid and consequently buoyancy force enhances the flow velocity further. In the lower half of the channel, the transfer of heat takes place from the fluid to the lower plate at z =0. Thus, the resultant velocity increases with increasing in *G*r. It is noticed that, the magnitude of the both velocity components *u* & *v* and the resultant velocity enhance with increase in *P* (Fig.10&11)). i.e., the increasing pressure gradient *P* leads to the increase of velocity. The variation of velocity profile with radiation parameter *R* is shown in (Fig. 12 & 13). The velocity *u* decreases and *v* increases with increasing the radiation parameter *R*. The resultant velocity also decreases with increasing in *R*.

From Fig. (14), we present the behaviour of the temperature profile, θ for various values of the radiation parameter *R*. It is observed that the temperature profile increases with minimum at the lower plate and maximum at the upper plate. However, a general decrease in the fluid temperature profile within the channel is observed with increase in the radiation parameter *R*.

The magnitude of the skin-friction τ_x increase with *M*, *D*, *E*, *Gr* and *P*, reduce with increase in R. Likewise the magnitude of τ_y increase with increase in *E*, *P* and *R*, reduce with increase in *M*, *D* and *Gr* (Tables 1-2). From Table 3, it is observed that the effect of increasing radiation parameter *R* and is to increase the magnitude of the rate of heat transfer (Nusselt number).

IV. Conclusions

It can be concluded that the fluid velocity profile is parabolic with maximum magnitude along the channel centre line and minimum at the walls. It is interesting to note that, the velocity of the fluid decreases with increases in the intensity of the magnetic field *M*, radiation parameter *R* and enhanced with increasing in Rotation parameter *E*, Grashof parameter *Gr*, pressure gradient *P* and porosity parameter *D*. The fluid temperature within the channel is observed with increase in the radiation parameter *R*. The magnitude of the skin-friction τ_x increase with *M*, *D*, *E*, *Gr* and *P*, reduce with increase in R. Likewise the magnitude of τ_y increase with increase in *E*, *P* and *R*, reduce with increase in *M*, *D* and *Gr*. We observed that the magnitude of the rate of heat transfer (Nusselt number) is increases with increase in radiation parameter *R*.

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Graphs and Tables:

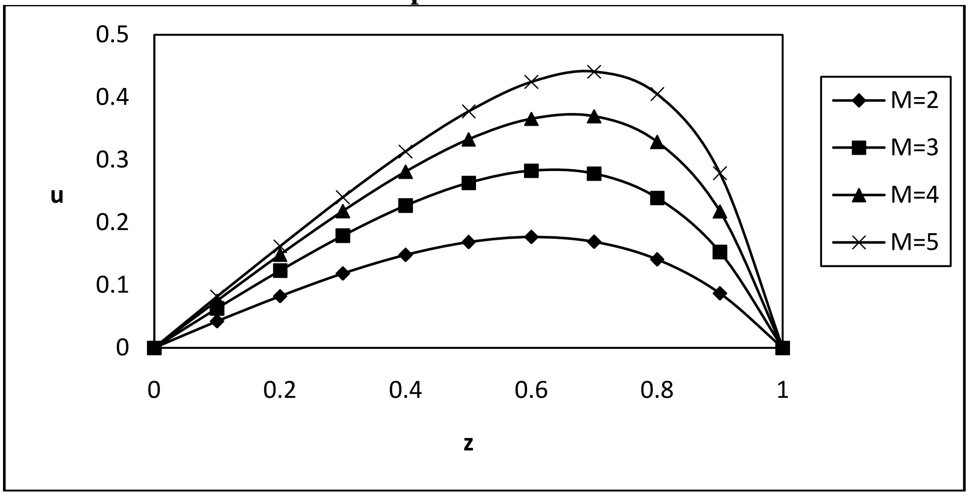

Fig. 2: The velocity Profile for *u* against *M* with $D = 0.2, E = 1, Gr = 1, P = 1, R = 1$

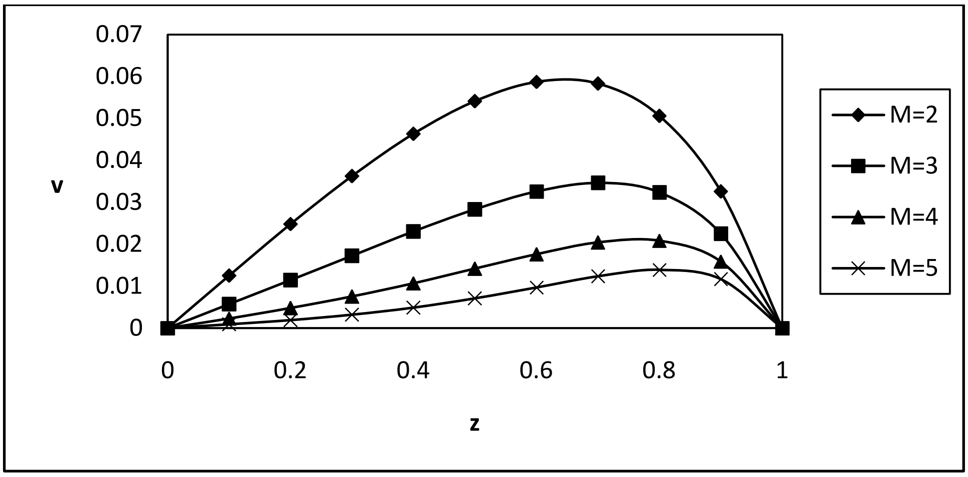

Fig. 3: The velocity Profile for *v* against M with $D = 0.2, E = 1, Gr = 1, P = 1, R = 1$

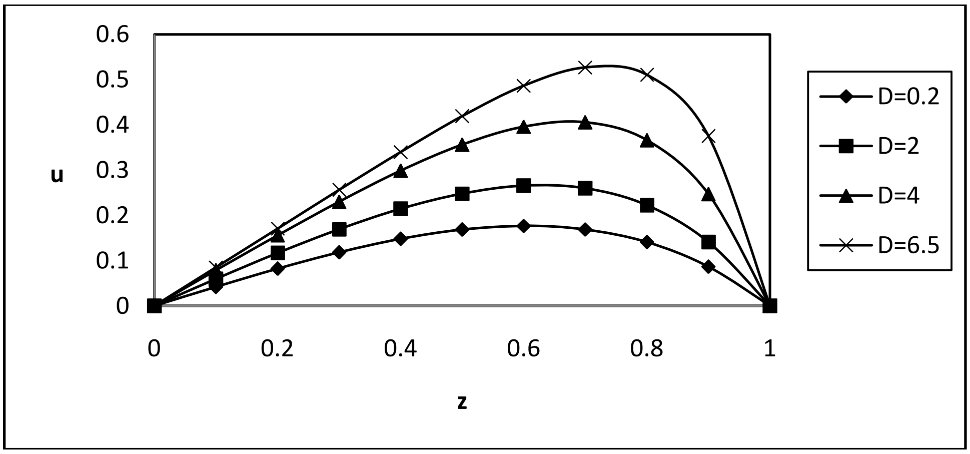

Fig.4: The velocity Profile for *u* against *D* with $M = 2, E = 1, Gr = 1, P = 1, R = 1$

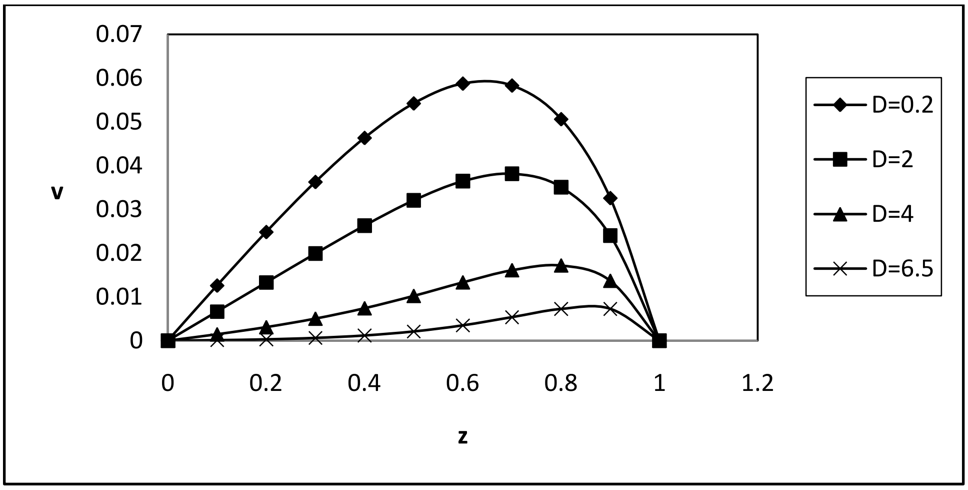

Fig. 5: The velocity Profile for *v* against *D* with $M = 2, E = 1, Gr = 1, P = 1, R = 1$

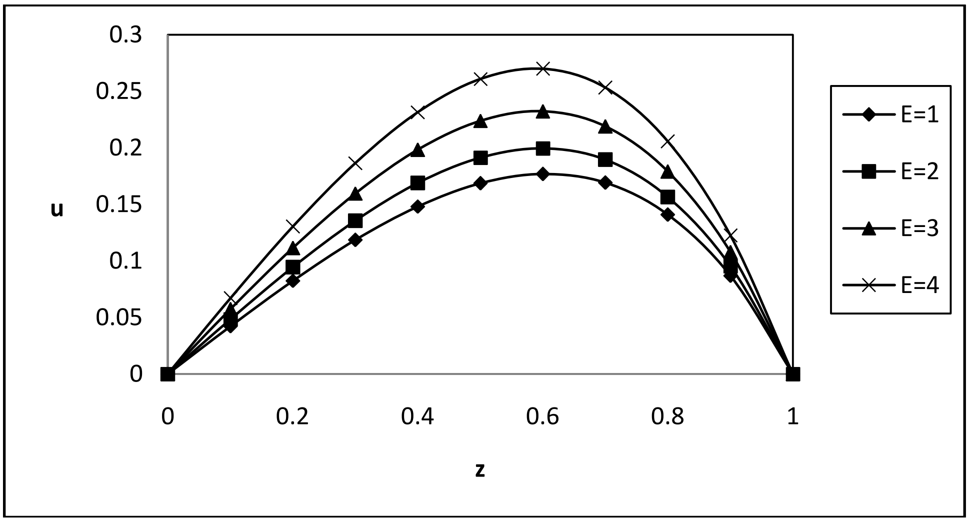

Fig. 6: The velocity Profile for *u* against *E* with $M = 2, D = 0.2, Gr = 1, P = 1, R = 1$

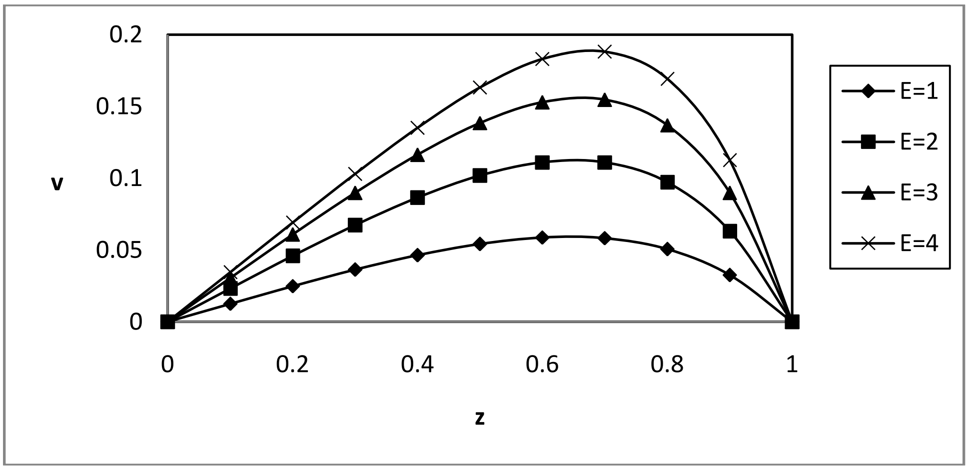

Fig. 7: The velocity Profile for *u* against *E* with $M = 2, D = 0.2, Gr = 1, P = 1, R = 1$

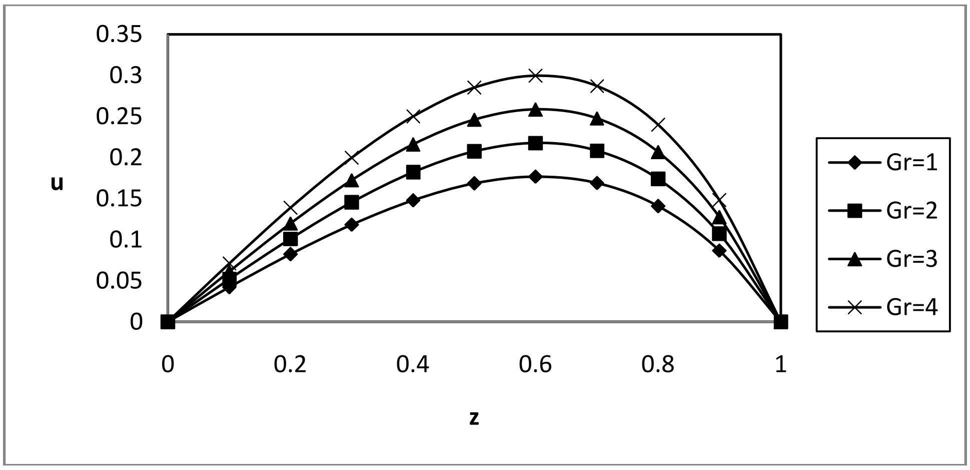

Fig. 8: The velocity Profile for *u* against *Gr* with $M = 2, D = 0.2, E = 1, P = 1, R = 1$

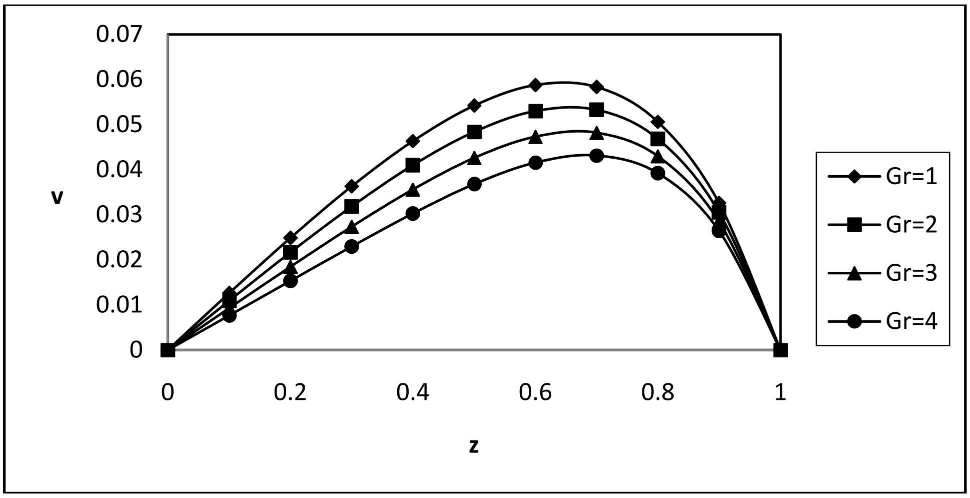

Fig. 9: The velocity Profile for *v* against *Gr* with $M = 2, D = 0.2, E = 1, P = 1, R = 1$

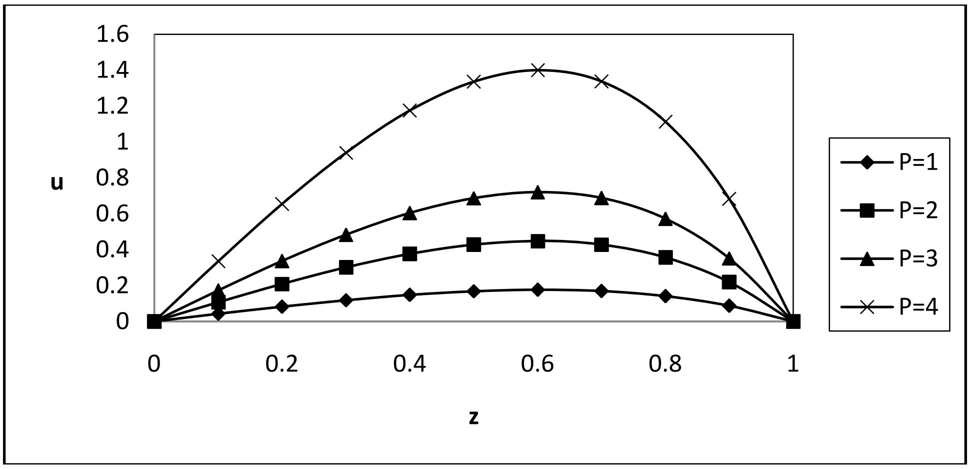

Fig. 10: The velocity Profile for *u* against *P* with $M = 2, D = 0.2, E = 1, Gr = 1, R = 1$

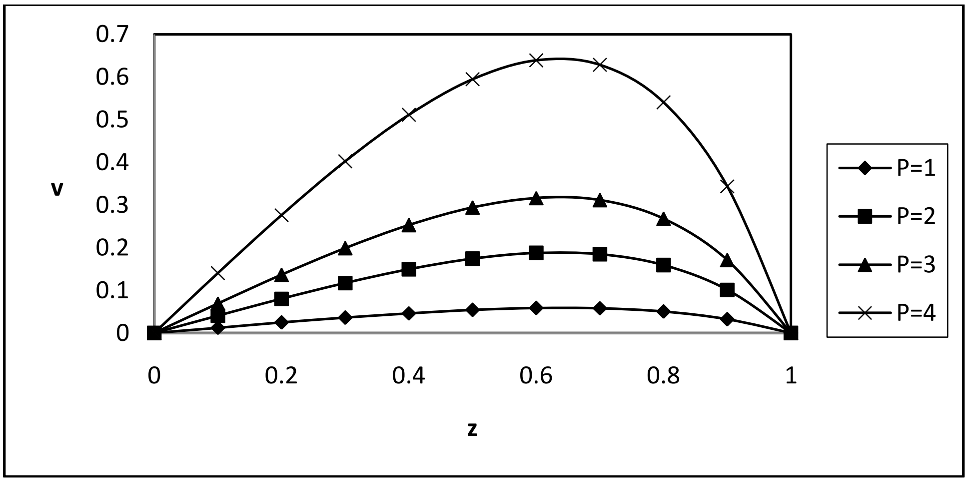

Fig. 11: The velocity Profile for *v* against *P* with $M = 2, D = 0.2, E = 1, Gr = 1, R = 1$

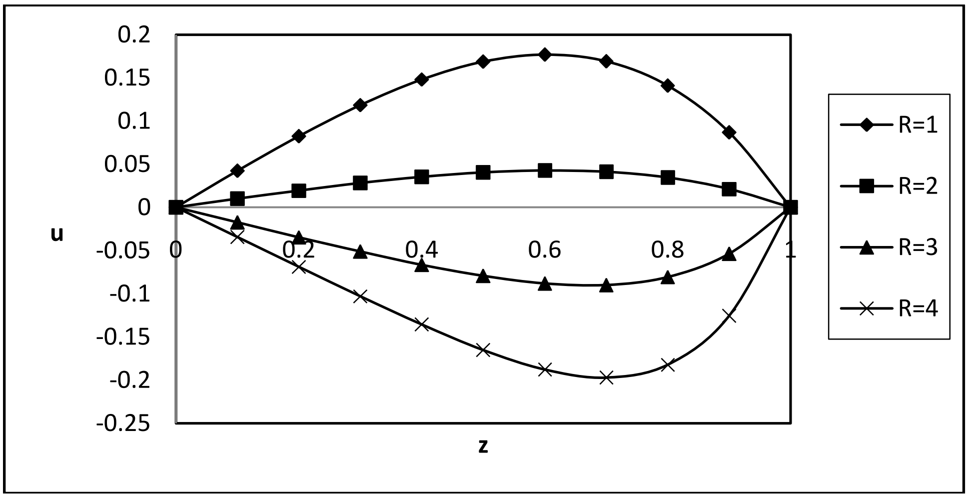

Fig. 12: The velocity Profile for *u* against *R* with $M = 2, D = 0.2, E = 1, Gr = 1, P = 1$

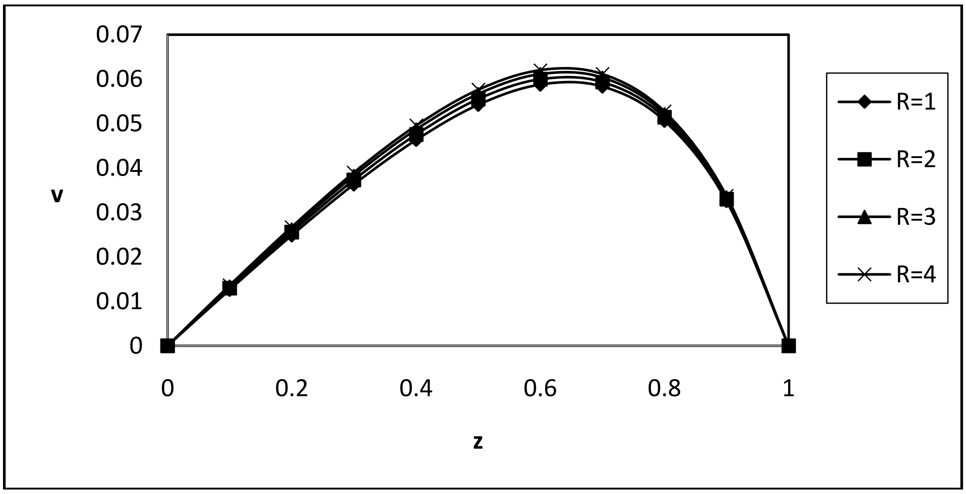

Fig. 13: The velocity Profile for *v* against *R* with $M = 2, D = 0.2, E = 1, Gr = 1, P = 1$

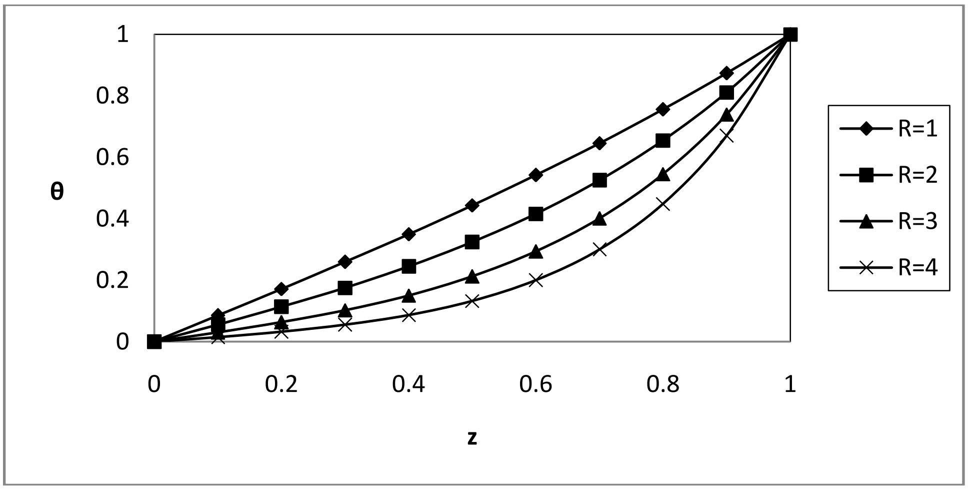

Fig. 14: The velocity Profile for θ against *R*

M	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**	**XI**
2	1.05515	1.09767	1.17505	1.14292	1.27208	1.30583	1.55651	1.85962	2.66409	0.26167	-0.7113
3	1.93532	1.96753	2.02656	1.97732	2.04304	2.14666	2.35800	3.65930	5.38328	1.14933	0.18378
4	2.88078	2.90591	2.95219	2.90146	2.93492	3.05948	3.23817	5.58288	8.28497	2.10067	1.14095
5	3.84871	3.86912	3.90681	3.85986	3.87819	4.00206	4.15540	7.54408	11.2394	3.07285	2.11740

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
D	0.2	0.5	0.8	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
E	1	1	1	2	3	1	1	1	1	1	1
Gr	1	1	1	1	1	2	3	1	1	1	1
P	1	1	1	1	1	1	1	2	3	1	1
R	1	1	1	1	1	1	1	1	1	2	3

Table. 1 The skin friction (τ_x) on upper plate

M	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**	**XI**
2	0.41571	0.40937	0.39823	0.80946	1.16658	0.39451	0.37331	0.85262	1.28953	0.41985	0.42389
3	0.31064	0.30767	0.30235	0.61413	0.90465	0.29825	0.28586	0.63368	0.95672	0.31295	0.31522
4	0.24082	0.23937	0.23675	0.47932	0.71336	0.23356	0.22629	0.48892	0.73702	0.24209	0.24336
5	0.19506	0.19428	0.19286	0.389278	0.58183	0.19061	0.18615	0.39458	0.59411	0.19579	0.19652

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
D	0.2	0.5	0.8	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
E	1	1	1	2	3	1	1	1	1	1	1
Gr	1	1	1	1	1	2	3	1	1	1	1
P	1	1	1	1	1	1	1	2	3	1	1
R	1	1	1	1	1	1	1	1	1	2	3

Table. 2 The skin friction (τ_y) on upper plate

	R=0.1	**R=0.3**	**R=0.5**	**R=0.9**
$\lvert Nu \rvert$	1.00333	1.02982	1.08198	1.25646

Table. 3 The Rate of Heat transfer (*Nu*) on upper plate